UNIT - I

CO-1 - Differential Calculus

1.If $\emptyset$ be the angle between the tangent and radius vector at any point on the curve $r = f(\theta)$ then $tan\emptyset$ equals to ________________

a)$\frac{dr}{ds}$ b)$r\frac{d\theta}{ds}$ c)$r\frac{d\theta}{dr}$ d) $\frac{d\theta}{dr}$

2.The angle between the radius vector and tangent for the vector $r = ae^{\theta cot\alpha}$ is _____________

a)$tan\alpha$ b)$cot\alpha$ c)α d)θ

3.The radius of curvature of the curve $y = e^x$ at the point where it crosses the y-axis is_______________

a)$2\sqrt{2}$ b)$\sqrt{2}$ c)2 d)$\frac{\sqrt{2}}{2}$

4)Curvature of a straight line is______________

a)∞ b)0 c)1 d)none of these

5)If the angle between the radius vector and the tangent is constant then the curve is ________

a)$r = acos\theta$ b)$r^2 = a^2cos^2\theta$ c)$r = ae^{b\theta}$ d) none of these

6)The curvature of the curve $x = acost\,, y = asint$ is

a)$\frac{\pi}{2}$ b) $\frac{a}{2}$ c)$\frac{\sqrt{\pi}}{2}$ d)a

7)The angle between the radius vector and tangent for the vector $r = a\theta$ is _____________

a)θ b)$\frac{1}{\theta}$ c)r d)$\frac{a}{\theta}$

8) The radius of curvature to the curve $x = at^2, y = 2at$ at the origin is ____________

a)$2a$ b)a c)2 d)$\frac{a}{2}$

9)The derivative of arc for the curve $y = f(x)$ is $\frac{ds}{dx}$ = _______________

a)$\sqrt{1 + (\frac{dy}{dx})^2}$ b)$1 + (\frac{dy}{dx})^2$ c) $1 + (\frac{dx}{dy})^2$ d)$\sqrt{1 + (\frac{dx}{dy})^2}$

10)The derivative of arc for the curve $x = f(y)$ is $\frac{ds}{dy}$=____________

a)$\sqrt{1+(\frac{dy}{dx})^2}$ b)$1+(\frac{dy}{dx})^2$ c) $1+(\frac{dx}{dy})^2$ d) $\sqrt{1+(\frac{dx}{dy})^2}$

11)The radius of curvature for the curve $x = e^t$, $y = e^{-t}$ at t=0 is ___________

a)$\frac{1}{\sqrt{2}}$ b)$\sqrt{2}$ c)2 d)$\frac{1}{2}$

12.The curvature of a function $f(x)$ is zero, which of the following functions could be $f(x)$?

a)$ax+b$ b)ax^2+bx+c c)$sinx$ d)$cosx$

13.The derivative of arc for the curve $x = f(t), y = g(t)$ is $\frac{ds}{dt}$ =_____________

a)$(\frac{dx}{dt})^2+(\frac{dy}{dt})^2$ b)$\frac{dx}{dt}+\frac{dy}{dt}$ c)$\sqrt{\frac{dx}{dt}+\frac{dy}{dt}}$ d)$\sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}$

14.The curvature of the function $f(x) = x^3 - x + 1$ at $x = 1$ is __________

a) $\frac{6}{5}$ b)$\frac{6}{5}$ c) $\frac{6}{5^{3/2}}$ d)$\frac{3}{5^{3/2}}$

15.The derivative of arc for the curve $r = f(\theta)$ is $\frac{ds}{d\theta}$=_____________

a) $\sqrt{(r^2+(\frac{dr}{d\theta})^2)}$ b)$\sqrt{(r^2+(\frac{d\theta}{dr})^2)}$ c)$\sqrt{(1+r^2(\frac{d\theta}{dr})^2)}$ d) $\sqrt{(1+r^2(\frac{dr}{d\theta})^2)}$

16.The derivative of arc for the curve $\theta = f(r)$ is $\frac{ds}{dr}$=_____________

a) $\sqrt{(r^2+(\frac{dr}{d\theta})^2)}$ b)$\sqrt{(r^2+((\frac{d\theta}{dr})^2)}$ c)$\sqrt{(1+r^2(\frac{d\theta}{dr})^2)}$ d) $\sqrt{(1+r^2(\frac{dr}{d\theta})^2)}$

17.The radius of curvature for the curve $y = f(x)$ is ρ =___________

a)$\frac{(1+y_2{}^2)^{3/2}}{y_1}$ b) $\frac{(1+y_1{}^2)^{3/2}}{y_2}$ c)$\frac{(1+y_1{}^2)^{2/3}}{y_2}$ d)$\frac{(1-y_1{}^2)^{3/2}}{y_2}$

18) The radius of curvature for the curve $x = f(t), y = g(t)$ is ρ =_____________

a) $\frac{(x'^2+y'^2)^{3/2}}{x'y''+y'x''}$ b)$\frac{(x'^2-y'^2)^{3/2}}{x'y''-y'x''}$ c) $\frac{(x'^2+y'^2)^{2/3}}{x'y''-y'x''}$ d)$\frac{(x'^2+y'^2)^{3/2}}{x'y''-y'x''})$

19)The radius of curvature for the curve $r = f(\theta)$ is ρ =____________

a) $\frac{(r^2+r_1{}^2)^{3/2}}{r^2+2r_1{}^2-rr_2}$ b)$\frac{(r^2+r_1{}^2)^{3/2}}{r^2-2r_1{}^2-rr_2}$ c)$\frac{(r^2-r_1{}^2)^{3/2}}{r^2+2r_1{}^2-rr_2}$ d)$\frac{(r^2+r_1{}^2)^{2/3}}{r^2+2r_1{}^2-rr_2}$

20)If the curvature of a curve increases then , the radius of curvature

a)increases b)decreases c)constant d)none of these

21)For the curve in polar form $\sqrt{\frac{r}{a}} = \sec\left(\theta/2\right)$ the value of $\frac{ds}{d\theta}$ is ___________

a) $rsec\theta$ b) $r\sec\left(\theta/2\right)$ c)$rsec2\theta$ d)$r\mathrm{cosec}\left(\theta/2\right)$

22)The angle between the radius vector $r = a(1 - cos\theta)$ and tangent to the vector is $\emptyset =$______________

a)$\frac{\theta}{2}$ b) θ c)0 d)$\frac{\pi}{2}$

23.For the polar curve $r = f(\theta)$, the relation between θ and coordinates (x,y) is________

a)$tan\theta = \frac{x}{y}$ b)$1 + sin\theta = \frac{y}{x}$ c)$1 + sec^2\theta = \frac{y^2}{x^2}$ d)$1 + cos\theta = \frac{x}{y}$

24. For the curve $alog(\sec\left(\frac{x}{a}\right))$ the value of $\frac{ds}{dx}$ is_____________

a)$cos\emptyset$ b)$sec\emptyset$ c) $tan\emptyset$ d)$cot\emptyset$

25.If the parametric equation of the curve is given by $x = ae^t sint$ and $y = ae^t cost$ then $\frac{ds}{dt}$ =___________

a)ae^t b)$2ae^t$ c)$\sqrt{3}ae^t$ d)$\sqrt{2}ae^t$

26) For the curve $y = x^2$ the value of $\frac{ds}{dx}$ at the point (1,1) is _________

a)$\sqrt{5}$ b)5 c)$\sqrt{4}$ d)4

27) For the curve $r\theta = a,$ $\frac{ds}{dr} =$___________

a)$\sqrt{1 - \theta^2}$ b)$\sqrt{1 + \theta^2}$ c)$\sqrt{(1 + \theta)^2}$ d)$\sqrt{(1 - \theta)^2}$

28) For the curve $= a(1 - cos\theta)$, $\frac{ds}{d\theta}$ is ________

a)$2acos\frac{\theta}{2}$ b)$2asin\frac{\theta}{2}$ c)$\sqrt{2}asin\frac{\theta}{2}$ d)$\sqrt{2}acos\frac{\theta}{2}$

29) For the curve $x^2 = y^3$, $\frac{ds}{dy}$ is ____________

a)$\sqrt{1 - \frac{9y}{4a}}$ b)$\sqrt{1 + \frac{9y}{4a}}$ c)$\sqrt{1 - \frac{9x}{4a}}$ d)$\sqrt{1 + \frac{9x}{4a}}$

30) The radius of curvature of the curve $y = x^2$ at the point (0,1) is _____

a)2 b) 1 c)1/2 d)1/4

31.Rolle’s theorem can be applied to $f(x) = x 2$ in the interval

i) [1 2] ii) [0 1] iii)[−1 1] iv) none of these

32. The value of c got by applying Rolle’s theorem to $f(x) = \sin x\ e\ x$ ($0 \leq x \leq \pi$) is

i) $\pi/2$ ii) $\pi/4$ iii) $\pi/3$ iv) none of these

33. If f(x) is differentiable for all $x \in (-\infty\ \infty)$ and if $x = a$ and $x = b(b > a)$ are two distinct real roots of $f(x) = 0$ then there exists

i) at least one value of $x \in [a\ b]$ such that $f\ 0\ (x) = 0$

ii) at least one value of $x \in [-\infty\ a]$ such that $f\ 0\ (x) = 0$

iii) at least one value of $x \in [b\ \infty]$ such that $f\ 0\ (x) = 0$

iv) none of these

34. If g(x) is differentiable for all $x \in (-\infty\ \infty)$, $g(a) = g(b)$ and if $f(x) = g(x)+ (x\ 2/2)$ then there exists

i) at least one fixed point of $f\ 0\ (x)$ in [a b]

ii) exactly one fixed point of $f\ 0\ (x)$ in [a b]

iii) no fixed point of $f\ 0\ (x)$ in [a b]

iv) none of these

35. If $f : R \to R$ is everywhere differentiable function such that $f(0) = f(1) = f(2)$ then there exists

i) at least two values of $x \in [0\ 2]$ such that $f\ 0\ (x) = 0$

ii) exactly two values of $x \in [0\ 2]$ such that $f\ 0\ (x) = 0$

iii) at most two values of $x \in [0\ 2]$ such that $f\ 0\ (x) = 0$

iv) none of these

36. If $f : R \to R$ is such that it is differentiable in [1 3], continuous in [2 4] and $f(1) = f(2) = f(3) = f(4)$ then there exists

i) at least one value of $x \in (3\ 4)$ such that $f\ 0\ (x) = 0$

ii) at least two values of $x \in (1\ 3)$ such that $f\ 0\ (x) = 0$

iii) at least two values of $x \in (2\ 4)$ such that $f\ 0\ (x) = 0$

iv) none of these

37. If $f : R \to R$ is such that it is differentiable in [1 2], continuous in [3 4] and $f(1) = f(2) = f(3) = f(4)$ then there exists

i) at least one value of $x \in (1\ 4)$ such that $f\ 0\ (x) = 0$

ii) at least three values of $x \in (1\ 4)$ such that $f\ 0\ (x) = 0$

iii) at least two values of $x \in (1\ 4)$ such that $f\ 0\ (x) = 0$

iv) none of these

38. If $g(x)$ is everywhere differentiable function such that $g(a) = g(b) = 0$ and if $f(x) = g(x) + x$ then there exists

i) at least one value of $x \in (a\ b)$ such that $f\ 0\ (x) = 1$

ii)at least one value of $x \in (a\ b)$ such that $g\ 0\ (x) = 3$

iii) at least one value of $x \in (a\ b)$ such that $f\ 0\ (x) = g\ 0\ (x)$

iv) none of these

39. If $f(x)$ is everywhere differentiable function such that $f(0) = 1, f(1) = 3$ and if $f(2) = 5$ then there exists

i) at least two values of $x \in (0\ 3)$ such that $f\ 0\ (x) = 2$

ii)exactly two values of $x \in (0\ 3)$ such that $f\ 0\ (x) = 2$

iii) at most two values of $x \in (0\ 3)$ such that $f\ 0\ (x) = 2$

iv) none of these

40. If $f : R \to R$ is such that it is differentiable in [2 4], continuous in [3 5] and $f(2) = 5$, $f(3) = 10$, $f(4) = 16$ and $f(5) = 25$ then there exists

i) at least one value of $x \in [4\ 5]$ such that $f\ 0\ (x) = 5$

ii) at least one value of $x \in [2\ 3]$ such that $f\ 0\ (x) = 5$

iii)at least one value of $x \in [3\ 4]$ such that $f\ 0\ (x) = 5$

iv) none of these

41. If $f : R \to R$ is differentiable in [1 3] and if $f(1) = 4$, $f(2) = 7$, $f(3) = 10$ and $f(4) = 13$ then there exists i) at least two values of $x \in [1\ 4]$ such that $f\ 0\ (x) = 3$

ii) at least three values of $x \in [1\ 4]$ such that $f\ 0\ (x) = 3$

iii)at most one value of $x \in [1\ 4]$ such that $f\ 0\ (x) = 3$

iv) none of these

42. The value of c got by applying Lagrange's mean value theorem to the function f(x) = x 2 in [0 4] is i) 1 ii) 2 iii) 3 iv) none of these 3

43. Lagrange's mean value theorem can be applied to the function f(x) =| x | in the interval

i) [−1 1] ii) [−2 1] iii) [1 2] iv) none of these

44. The value of c got by applying Cauchy's mean value theorem for the functions f(x) = e x and g(x) = e −x in [0 1] is

i) 1/2 ii) 2/3 iii) 1/3 iv) none of these

45. Cauchy's mean value theorem can be applied to the functions f(x) = x 3 − 2x 2 and g(x) = x 2 in the interval

i) [−1 1] ii) [−2 1] iii) [2 3] iv) none of these

46. If f, g : R → R are everywhere differentiable functions such that f 0 (x) 6= 0 in (a b), f(a) = g(a) and f(b) = g(b) then there exists

i) at least one value of x ∈ [a b] such that f 0 (x) = g 0 (x)

ii) at most one value of x ∈ [a b] such that f 0 (x) = g 0 (x)

iii) no value of x ∈ [a b] such that f 0 (x) = g 0 (x)

iv) none of these

47. If f, g : R → R are functions such that f(n) = g(n) for all n ∈ N, f 0 (x) exists in [1 4] and g 0 (x) > 0 in [2 5] then there exists

i) at least one value of x ∈ [1 5] such that f 0 (x) = g 0 (x)

ii) at most one value of x ∈ [1 5] such that f 0 (x) = g 0 (x)

iii) no value of x ∈ [1 5] such that f 0 (x) = g 0 (x)

iv) none of these

48. If f : [1 4] → R is a differentiable function and if f(2) = f(3) then

i) f (c) = 0 for some c ∈ (1 4) ii) f (c) = 0 for some c ∈ (1 2)

iii) f (c) = 0 for some c ∈ (3 4) iv) none of these

49. If f : [0 5] → R is a differentiable function then

i) f (c) = f(2)−f(1) for some c ∈ (1 2)

ii) f(c) = f(3)−f(1) for some c ∈ (1 3)

iii) $f(c) = f(4) - f(1)$ for some $c \in (1\ 4)$

iv) none of these

50. If $f, g : [0\ 3] \to R$ are differentiable functions such that $g\ 0\ (x) \geq 0$ in $(1\ 2)$ then 4

i) $f\ (c) / g\ (c) = (f(2) - f(1)) / (g(2) - g(1))$ for some $c \in (1\ 2)$

ii) $(f\ (c) / g\ (c) = (f(3) - f(2)) / (g(3) - g(2))$ for some $c \in (2\ 3)$

iii) $f\ (c) / g\ (c) = (f(1) - f(0)) / (g(1) - g(0))$ for some $c \in (0\ 1)$

iv) none of these

Answer: 1) c 2)c 3)a 4)b 5)c 6)a 7) a 8)b 9) a 10)d 11)b 12)a 13)d

14)d 15)a 16) c 17)b 18)d 19)a 20)b 21)b 22)a 23)c 24)d 25)d 26)a 27)b 28) b 29)b 30) c 31) iii 32) ii 33) i 34) i 35) i 36) ii 37) i 38)i 39)i 40) ii 41) i 42) ii 43) iii 44) i 45) iii 46) i 47) i 48) i 49) i 50) iv